

ANALYSIS OF THE EFFECT OF INITIAL CONDITIONS
ON THE INITIAL DEVELOPMENT OF A TURBULENT JET

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P. 6

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The effect of the initial condition at the jet exit on the downstream evolution, particularly within the potential core length, were numerically investigated as well as with available experimental data. In order to select most dependable computational model for the present numerical experiment, a comparative study has been performed with different turbulence models at k - ϵ level, and it was found that the k - ϵ - γ model yields superior prediction accuracy over other conventional models. The calculated results show that the potential core length and the spreading rate the initial mixing layer are dependent on the initial length scale as well as the turbulent kinetic energy at the jet exit. Such effect of the initial length scale increases with higher initial turbulence level. An empirical parameter has been devised to collapse the calculated data of the potential core length and the spreading rate with various initial conditions onto a single curve.

Introduction

It is well known that the potential core length, the spreading rate and the asymptotic peak turbulence intensity vary widely from experiment to experiment in the jet initial region.^{1,2} Husain and Hussain³ showed experimentally that the boundary layer state, laminar or turbulent, the momentum thickness and the fluctuation level in the initial boundary layer at the jet exit were important factors which govern the downstream jet development. Gutmark and Ho⁴ found that such scatter of the experimental data stems from spatially coherent disturbances in individual facilities. They considered the initial instability frequency as one of the important initial conditions affecting the jet evolution.

In addition to the condition of the initial boundary layer, however, since the jet exit flow field is composed of the boundary layer near the inner wall and the core flow in the central region, the turbulent state of the initial jet core must also affect the downstream jet evolution process. Turbulent intensity in a laboratory jet is typically 0.5% or less, while those in practical turbojet and turbofan engines have been reported to be between 3% and 15%.⁵ Thus, in the initial region of the jet flow, the mixing layer and the turbulent core should interact with each other. If the level of the initial core turbulence is low, the effect of the interaction may be small or negligible. However, if it is sufficiently high, the flow field in the initial region should be regarded as a complex flow according to Bradshaw's category.⁶

Vlasov et al.⁷ reported that the potential core length significantly decreases with increasing initial core turbulence. More elaborate experiment was performed by Raman et al.⁵ who kept the exit mean velocity profile and the boundary layer state nearly the same, but varied the core turbulent intensity between 0.15% and 5% by using various turbulence generating grids. From the variation of the mean velocity along the jet centerline, they concluded that the turbulent intensity in the initial core has only small effect on the jet evolution. However, considering that the freestream length scale is an important parameter for the development of the turbulent boundary layer, which has been vividly demonstrated by Hancock and Bradshaw⁸, the length scale of the core turbulence should be considered as an additional controlling parameter for the downstream jet development. Unfortunately, however, experimental data of the initial length scale or dissipation rate are almost unavailable from published reports. Therefore, in the present study, a computational analysis is carried out to systematically investigate the effects of the turbulent intensity and the length scale in the initial core region on the initial development of a turbulent jet flow.

Since most previous computational studies have been concentrated on the flow field in the self-preserving region, those on the jet initial region are only scarcely found in open literatures. Islam and Tucker⁹ computed the turbulent flow of a jet initial region by a revised mixing length model. Meanwhile, computational turbulence models such as k - ϵ and Reynolds stress models have serious "anomaly problems" when they are applied to compute turbulent free shear flows: "round-jet / plane-jet anomaly"¹⁰ and "plane-wake / plane-jet anomaly"¹¹. Recently, Cho and Chung¹² developed a new k - ϵ - γ model and made considerable improvement in the prediction accuracy for free shear flows in their similarity regions.

In the present study, firstly, three variants of k - ϵ model and the new k - ϵ - γ model were applied to the initial region of the round and plane jets to prove that the k - ϵ - γ model is more reliable than other models. Secondly, using the k - ϵ - γ model the effects of the initial core turbulence, i.e. the turbulent kinetic energy, and the dissipation rate or length scale are systematically investigated, and the results are compared with available experimental data in the initial jet region.

Computational Models

In order to numerically examine the initial jet evolution process which exhibits quite complex nature of turbulence, a dependable computational model must be employed. As is well known, the k - ϵ model has a number of variant forms which has been formulated to remedy the vulnerable model coefficients of the standard k - ϵ model under certain circumstances. One of such weaknesses in computation of free shear flows has been expressed by a term "round-jet / plane-jet anomaly"¹⁰. Specifically, the predictions of a round jet and a plane jet with the same model constants show inconsistent results: If the model constants are adjusted with reference to the spreading rate of the plane jet, the computed spreading rate of the round jet is higher than that of the plane jet by as much as 25% whereas most experimental data demonstrate that the round jet spreads slower than the plane jet by about 15%.

Pope¹⁰ attributed the anomaly to the neglect of the mean vortex stretching effect in the source term of the dissipation equation, and introduced a vortex stretching invariant term $\chi \equiv (k/\epsilon)^3 \Omega_{ij} \Omega_{kl} S_{ij}$, where Ω_{ij} and S_{ij} are the rate of mean rotation and rate of mean strain tensors, respectively. Note that the invariant χ has a positive value in the round jet whereas it vanishes in the plane jet. Thus, the modified form of k - ϵ model suggested by Pope is as follows:

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$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_j} \left[\frac{v_i}{\sigma_k} \frac{\partial k}{\partial x_j} \right] + P - \epsilon \quad (1)$$

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial x_j} \left[\frac{v_i}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_j} \right] + \frac{\epsilon^2}{k} \left[C_{\epsilon 1} \frac{P}{\epsilon} - C_{\epsilon 2} + C_{\epsilon 3} \lambda \right] \quad (2)$$

$$v_i = c_\mu \frac{k^2}{\epsilon} \quad P = -\overline{u_i u_j} \frac{\partial U_i}{\partial x_j}$$

Here the model constants are ; $C_\mu=0.09$, $\sigma_k=1.0$, $\sigma_\epsilon=1.3$, $C_{\epsilon 1}=1.45$, $C_{\epsilon 2}=1.90$, $C_{\epsilon 3}=0.79$.

Hanjalic and Launder¹³ found that the rate of spectral energy transfer across the wave number space, which is nearly equal to the dissipation rate, is significantly promoted by irrotational deformation which is associated with normal strains. They also noted that the irrotational deformation has larger value in the round jet than in the plane jet, which stimulated them to propose the following variant of the $k-\epsilon$ model to solve the "round-jet / plane-jet anomaly" problem.

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_j} \left[\frac{v_i}{\sigma_k} \frac{\partial k}{\partial x_j} \right] + P_{k,s} + P_{k,n} - \epsilon \quad (3)$$

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial x_j} \left[\frac{v_i}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_j} \right] + \frac{\epsilon^2}{k} \left[C_{\epsilon 1} \frac{P_{k,s}}{\epsilon} - C_{\epsilon 2} + C_{\epsilon 3} \frac{P_{k,n}}{\epsilon} \right] \quad (4)$$

$$P_{k,s} = -\overline{u_i u_j} \frac{\partial U_i}{\partial x_j} \quad (i \neq j), \quad P_{k,n} = -\overline{u_i u_j} \frac{\partial U_i}{\partial x_j} \quad (i=j)$$

$$\overline{u^2} - \overline{v^2} = c_{uv} k, \quad v_i = c_\mu \frac{k^2}{\epsilon}$$

where $C_\mu=0.09$, $\sigma_k = \sigma_\epsilon=1.0$, $C_{\epsilon 1}=1.44$, $C_{\epsilon 2}=1.90$, $C_{\epsilon 3}=4.44$, $C_{uv}=0.33$.

Quite recently, Cho and Chung¹² showed that, although the above two variants improve the consistency in predicting the plane jet and round jet with varying degree of accuracy, such modifications do not yield any better solution to the "plane-jet / plane-wake anomaly problem" which was raised as another computational anomaly through AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows in 1980.¹¹ With a lengthy discussion about the role of intermittency in the mixing layer between the shear flow in the core region and the ambient potential flow outside the jet boundary, they proposed a new $k-\epsilon-\gamma$ equation model as follows (See Cho and Chung¹² for details) :

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_j} \left[\frac{v_i}{\sigma_k} \frac{\partial k}{\partial x_j} \right] + P_{k,s} + P_{k,n} - \epsilon \quad (5)$$

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial x_j} \left[\frac{v_i}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_j} \right] + \frac{\epsilon^2}{k} \left[C_{\epsilon 1} \frac{P_{k,s} + 3P_{k,n}}{\epsilon} - C_{\epsilon 2} + C_{\epsilon 3} \lambda + C_{\epsilon 4} \Gamma \right] \quad (6)$$

$$\frac{D\gamma}{Dt} = \frac{\partial}{\partial x_j} \left[(1-\gamma) \frac{v_i}{\sigma_\gamma} \frac{\partial \gamma}{\partial x_j} \right] + c_{\beta 1} \gamma (1-\gamma) \frac{P_{k,s} + P_{k,n}}{k} + c_{\beta 2} \frac{k^2}{\epsilon} \frac{\partial \gamma}{\partial x_j} \frac{\partial \gamma}{\partial x_j} - c_{\beta 3} \gamma (1-\gamma) \frac{\epsilon}{k} \Gamma \quad (7)$$

$$v_i = c_\mu \left(1 + C_{\mu 8} \frac{k^3}{\epsilon^2} \frac{1-\gamma}{\gamma^3} \frac{\partial \gamma}{\partial x_k} \frac{\partial \gamma}{\partial x_k} \right) \frac{k^2}{\epsilon}$$

$$\Gamma = \frac{k^{2.5}}{\epsilon^2} \frac{U_i}{(U_k U_k)^{0.5}} \frac{\partial U_i}{\partial x_j} \frac{\partial \gamma}{\partial x_j}$$

Proposed model constants are; $C_\mu=0.09$, $C_{\mu 8}=0.1$, $\sigma_k = \sigma_\epsilon = \sigma_\gamma=1.0$, $C_{\epsilon 1}=1.44$, $C_{\epsilon 2}=1.92$, $C_{\epsilon 3}=0.30$, $C_{\epsilon 4}=0.10$, $C_{\beta 1}=1.6$, $C_{\beta 2}=0.15$, $C_{\beta 3}=0.16$.

For more detail computation, the Reynolds stress model may be utilized. However, it has been widely demonstrated that when it comes to compute the simple free shear flows, the Reynolds stress model yields similar prediction accuracy as the $k-\epsilon$ model¹⁴, and no attempt has been made at modifying the Reynolds stress model to solve the anomaly problem. For this reason, it was not included in the present numerical investigation

Initial Conditions and Computational Method

It is assumed that the velocity profile at the exit consists of two regions: a boundary layer near the inner wall and the core layer in the central region. The initial boundary layer is further assumed to be in a fully developed turbulent state. Thus, all turbulent parameters in that region can be estimated by those of a fully developed turbulent boundary layer over a flat plate. In practice, Husain and Hussain showed that the mean velocity and the turbulent intensity profiles in the initial boundary layer at the jet exit are close to the flat plate data. Therefore, we picked up the mean velocity profile and the turbulent kinetic energy profile from Klebanoff's experiment on a flat plate. And the dissipation rate data were calculated by assuming a local equilibrium. There have been a large number of jet experiments, however, unfortunately, we can not find any experiment which measured the initial levels of the turbulent kinetic energy and the dissipation rate in the core region, simultaneously. Therefore, we are managed to assume them within a physically reasonable range.

In the core region at the jet exit, the mean velocity, the turbulent kinetic energy and the dissipation rate were assumed uniform, but with their magnitudes being different for different cases. In order to specify the relative magnitude between the turbulent kinetic energy and the dissipation rate, i.e. the initial eddy viscosity level k^2/ϵ , in a physically realistic range, the data from a grid-generated turbulence were adopted. Comte-Bellot and Corrsin¹⁵ presented various data set of the energy decay of the grid turbulence. Fig.1 represents the relations between the length scale and the intensity of turbulence for three cases in Comte-Bellot and Corrsin. From these relations, a total of 20 pairs of data were used to specify the initial turbulent kinetic energy and the dissipation rate in the core region at the jet exit. Since the boundary layer profiles are nearly invariant within 50% of the boundary layer thickness, δ , the initial profiles of the mean velocity, the turbulent kinetic energy and the dissipation rate are smoothly connected in the region $0.5\delta-\delta$.

The upwind finite-difference procedure¹⁶ was used to solve the system of the governing equations. Predictions of the jet flow reported below were obtained by using 200 cross-stream nodes, 50 uniform nodes inside the jet exit diameter and 150 stretched nodes outward. The jet exit mean velocity U_e was 20 m/sec and jet exit diameter D or width H was 10 cm, hence, Reynolds number based on D or H was about 1.3×10^5 . Initial boundary layer thickness and the momentum thickness were assumed 6mm and 1mm, respectively. The turbulent kinetic

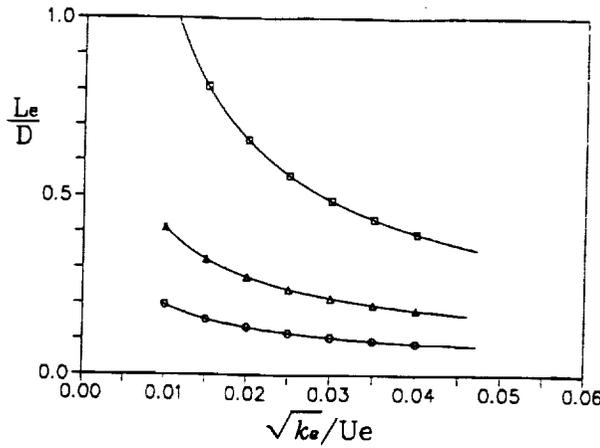


Fig.1 The initial conditions of the jet exit core region selected from Comte-Bellot and Corrsin : \circ ; $R_M=3.4 \times 10^4$, Δ ; $R_M=6.8 \times 10^4$, \square ; $R_M=13.5 \times 10^4$.

energy and the dissipation rate were estimated by assuming $u^* = 1.0$ m/sec.

Results and Discussion

Performance Tests of Computational Models

In the present investigation, the standard $k-\epsilon$ model¹⁷, modified $k-\epsilon$ models by Pope, and Hanjalic and Launder, and the $k-\epsilon-\gamma$ model were applied to compute the initial flow field of a plane jet and a round jet for a case with $\sqrt{k_e}/U_e = 0.01$, $Le/D = 0.2$ in Fig.1. Table.1 represents the predicted potential core lengths and spreading rates. The spreading rate can be

defined in various ways. The shear layer width at a certain downstream location x is determined by either $B_1 = y_{0.1} - y_{0.9}$ or $B_2 = y_{0.1} - y_{0.95}$, and the vorticity thickness δ_w defined by $-Ue/(\partial U/\partial y)_{max}$, where $y_{0.1}$, $y_{0.9}$ and $y_{0.95}$ indicate cross-stream locations from the jet centerline where the local mean velocity is 10%, 90% and 95% of the centerline mean velocity, respectively. The symbol x_p represents the potential core length.

By comparing the predicted values in Table.1, it is concluded that the $k-\epsilon-\gamma$ model provides the most reasonable predictions for all jet parameters. Specifically, the prediction of the potential core length is remarkably improved, which can also be appreciated from Fig.2 and 3. In Fig.3, the experimental data show that the turbulent kinetic energy at the jet centerline increases monotonically in the potential core region. However, all models failed to reproduce such increase. From the exact turbulent kinetic energy equation, it can be seen that, since there is no mean shear in the potential core, the turbulent kinetic energy should simply decay. Thus, it is likely that either a certain unknown mechanism exist in the core region or the real flow had some initial shear at the jet exit. Hussain and Husain²⁰ explained that this occurs because the core potential fluid is exposed to a 'massaging' effect of motions in the mixing layer all around of, which argument however cannot be supported by the governing field equation. Nevertheless, the $k-\epsilon-\gamma$ model predicts very fairly the variation of the turbulent kinetic energy along the centerline except in the potential core region.

Fig.4 represents the mean velocity profile in similarity coordinate at about the end of the potential core region. Before the end of the potential core region, the initial mixing layer attains similarity. This can be further clarified by the fact that the shear layer thickness varies linearly³. In all computations of the mean velocity, the turbulent shear stress

Table 1 Potential core lengths and initial spreading rates of jet flows

($\sqrt{k_e}/U_e = 0.01$, $Le/D = Le/H = 0.2$)

Flow	Model and experiment	x_p	$\frac{dB_1}{dx}$	$\frac{dB_2}{dx}$	$\frac{d\delta_w}{dx}$
round jet	$k-\epsilon-\gamma$	4.57 D	0.163	0.175	0.141
	Hanjalic and Launder's $k-\epsilon$	8.33 D	0.154	0.158	0.076
	Pope's $k-\epsilon$	7.89 D	0.146	0.152	0.112
	Standard $k-\epsilon$	7.21 D	0.155	0.162	0.128
	experiment	4.90 D ¹⁸	0.16 - 0.165 ¹	0.158 - 0.202 ²	0.112 - 0.175 ⁴
plane jet	$k-\epsilon-\gamma$	4.80 H	0.163	0.177	0.155
	Hanjalic and Launder's $k-\epsilon$	10.10 H	0.163	0.168	0.110
	Standard $k-\epsilon$	8.74 H	0.151	0.159	0.136
	experiment	4.50 H ¹⁹	0.155 - 0.180 ¹	-	0.155 - 0.179 ¹

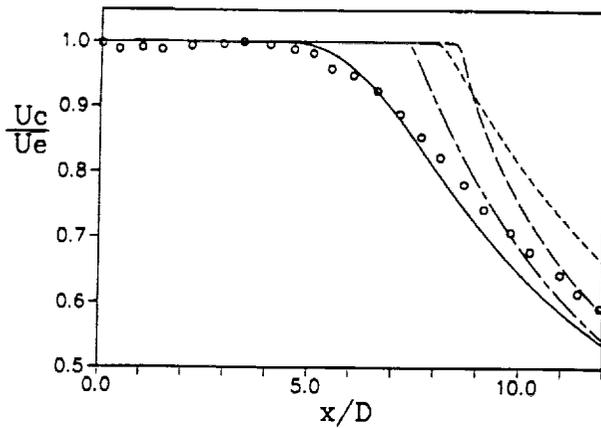


Fig. 2 The comparison of model on the variation of the streamwise mean velocity along the centerline in a roundjet : experimental data : \circ ; Raman et al. : predictions for $\sqrt{k}/U_c=0.01$, $L/D=0.2$: predictions : — ; $k-\epsilon-\gamma$ model of Cho and Chung ; - - - ; $k-\epsilon$ model of Hanjalic and Launder, - - - - ; $k-\epsilon$ model of Pope, - . - . ; standard $k-\epsilon$ model.

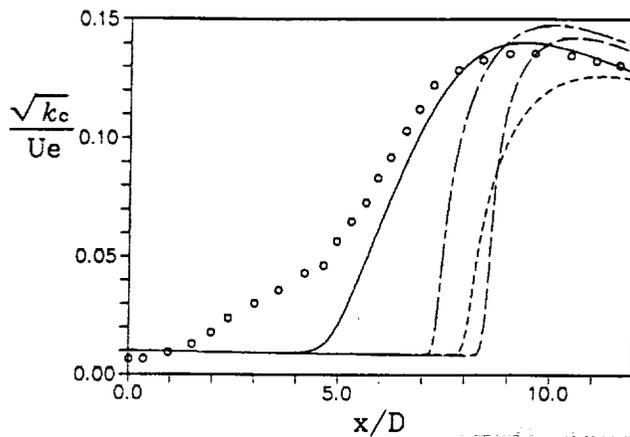


Fig. 3 The comparison of models on the variation of the turbulent kinetic energy along the centerline in a roundjet : experimental data : \circ ; Raman et al. for $u'/U_c=0.5\%$, assume $v'=w'=0.6u'$: predictions for $\sqrt{k_c}/U_c=0.01$, $L_c/D=0.2$, lines the same as Fig. 2.

and the turbulent kinetic energy, the $k-\epsilon-\gamma$ model outperforms over the $k-\epsilon$ models as can be seen in Figs. 4, 5 and 6. The better performance of $k-\epsilon-\gamma$ model in the core region ($(y-y_{0.5})/(y_{0.1}-y_{0.9}) < 0$) may be attributed to the correct representation of the interaction between the mean velocity gradient and the intermittency gradient by the $k-\epsilon-\gamma$ model (see, for details, Cho and Chung).

Effects of the Initial Conditions on the Downstream Evolution

In order to investigate the effects of the initial conditions on the jet downstream development, the $k-\epsilon-\gamma$ model was utilized. The initial conditions for the present computation were

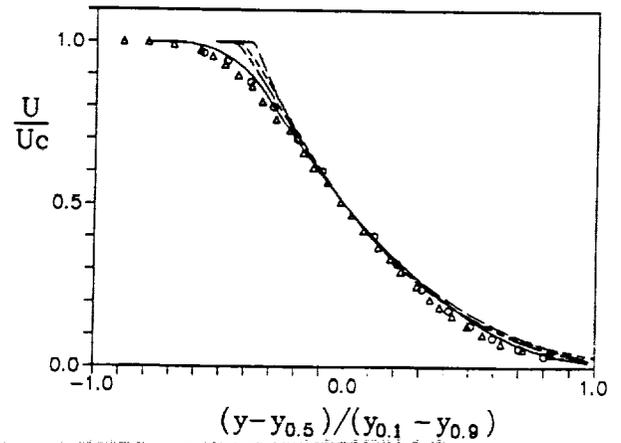


Fig. 4 The streamwise mean velocity profiles in the initial similarity region of a round jet : experimental data : \circ ; Bradshaw et al., Δ ; Husain and Hussain : predictions : — ; $k-\epsilon-\gamma$ model of Cho and Chung ; - - - ; $k-\epsilon$ model of Hanjalic and Launder, - - - - ; $k-\epsilon$ model of Pope, - . - . ; standard $k-\epsilon$ model.

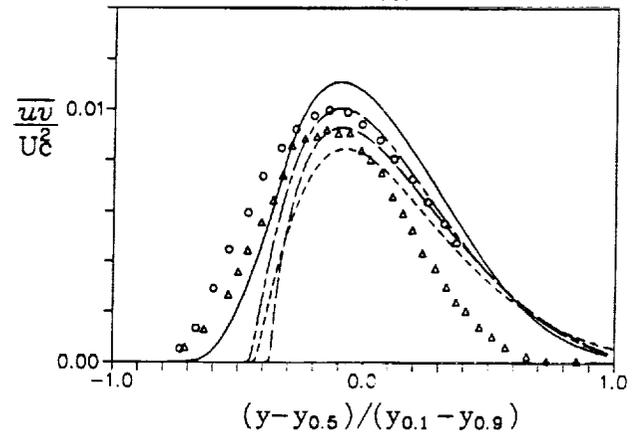


Fig. 5 The Reynolds shear stress profiles in the initial similarity region of a round jet : symbols and lines the same as Fig. 4.

selected from Fig. 1 as discussed previously. Fig. 7 reveals that the potential core length is smaller for higher initial turbulence level, but that the centerline mean velocity decay rates after the core region are nearly the same for all cases. Computed k variations along the centerline in Fig. 8 agree well with experimental data only after the core region. For initially high turbulence level, experimental data of the turbulent kinetic energy decay near the exit and then increase monotonically, but the computed one decays continuously in the potential core. The discrepancy between these two observations is not yet understood.

The variations of the potential core length and the spreading rate with different initial conditions are represented in Fig. 9. If the level of the initial turbulent kinetic energy is increased, the potential core length is reduced and the

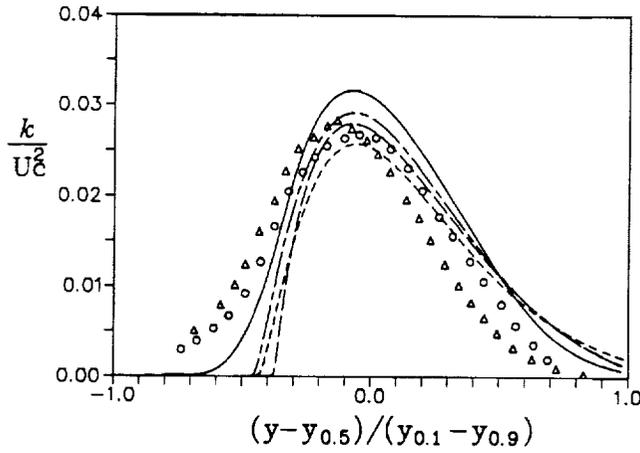


Fig.6 The turbulent kinetic energy profiles in the initial similarity region of a round jet : symbols and lines the same as Fig.4.

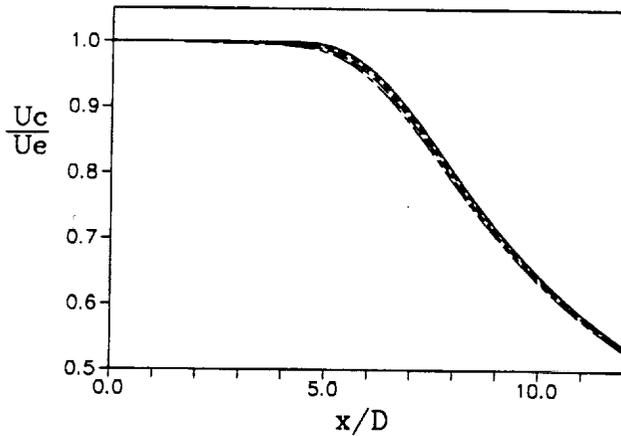


Fig.7 The variation of the streamwise mean velocity along the centerline in a round jet with the kinetic energy of the exit core turbulence for $R_M=3.4 \times 10^4$ in Fig.1 : \circ ; $\sqrt{k_c}/U_e=0.01$, Δ ; $\sqrt{k_c}/U_e=0.02$, \square ; $\sqrt{k_c}/U_e=0.03$, \diamond ; $\sqrt{k_c}/U_e=0.04$.

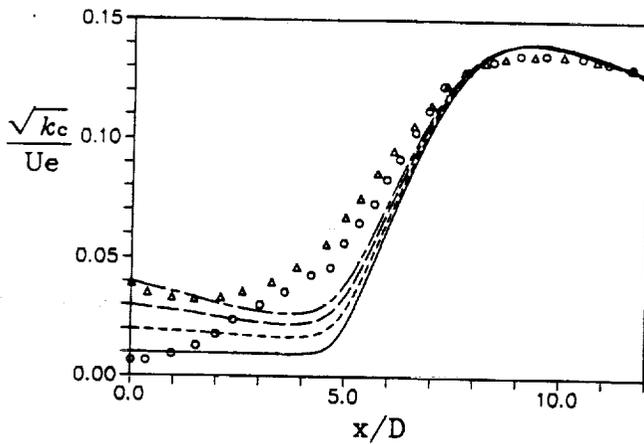


Fig.8 The variation of the turbulent kinetic energy along the centerline in a round jet with the kinetic energy of the exit core turbulence : experimental data from Raman et al. : \circ ; $u'/U_e=0.5\%$, Δ ; $u'/U_e=5.0\%$: prediction for $R_M=3.4 \times 10^4$ in Fig.1 , lines the same as Fig.7.

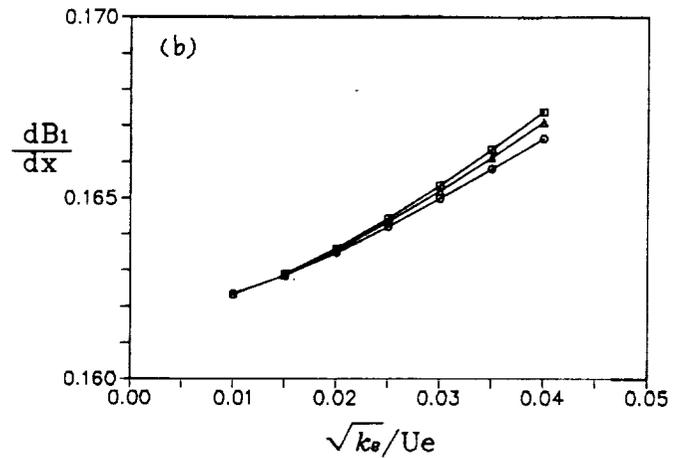
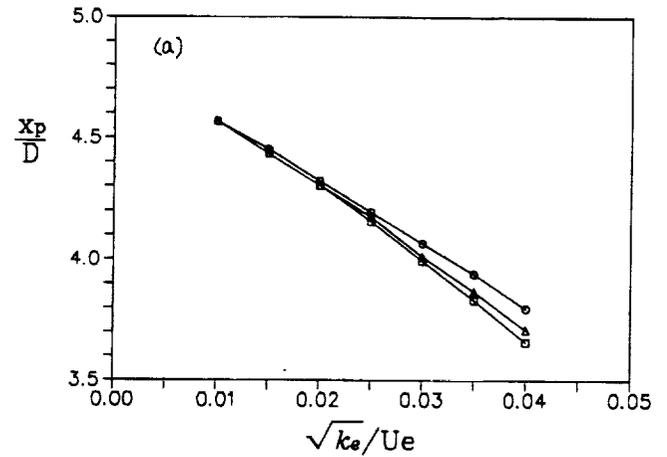


Fig.9 The variation of the predicted potential core length and the spreading rate in a round jet with initial conditions as a function of initial turbulence level : \circ ; $R_M=3.4 \times 10^4$, Δ ; $R_M=6.8 \times 10^4$, \square ; $R_M=13.5 \times 10^4$: (a) the potential core length ; (b) the spreading rate.

spreading rate becomes large. Also, it can be seen that the effect of the initial length scale is such that increasing initial length scale shortens the potential core length and augments the spreading rate. Moreover, such effect of the initial length scale is magnified at increased initial turbulence level. Consequently, the mixing is promoted by increasing both the initial turbulent kinetic energy and the initial length scale. This is because larger core length scales penetrate further into the mixing layer. Similar conclusion can be drawn from the experiment of Hancock and Bradshaw⁸, who carried out an experiment of the effects of freestream turbulence on a flat plate boundary layer.

Finally, an attempt is made at devising by trial-and-error to collapse the calculated data into a single correlation. The parameter found in this way is shown in Figs. 10(a) and (b), where the number 80 is an empirically determined constant. All data nicely fall on a single curve as can be seen in figures. This parameter was found to correlate the plane jet data too (not shown in this paper).

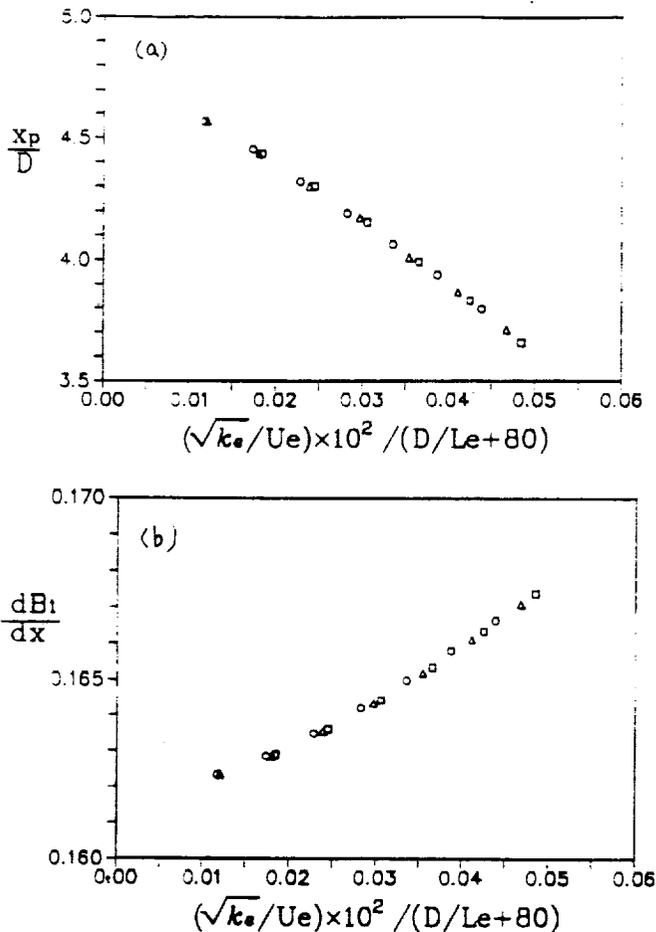


Fig.10 The variation of the predicted potential core length and the spreading rate in the initial region of a round jet with initial condition as a function of a nondimensional parameter : symbols the same as Fig.9 : (a) the potential core length ; (b) the spreading rate.

Conclusions

The effects of initial conditions at the jet exit have been numerically investigated. As a most reliable computational model, the $k-\epsilon-\gamma$ turbulence model has been selected by comparing the prediction accuracies of various turbulence models at $k-\epsilon$ level. It was found that the standard form and a couple of variants of the $k-\epsilon$ model yield too lengthy potential core and lower spreading rate, whereas the $k-\epsilon-\gamma$ model reproduce faithfully the turbulent flow field in the jet initial region.

The calculated results show that the potential core length and the spreading rate in the initial mixing layer are dependent on the initial length scale as well as on the turbulent kinetic energy at the jet exit. Such effect of the initial length scale increases with higher initial turbulence level. An empirical parameter has been devised to collapse the calculated data of the potential core length and the spreading rate with various initial conditions onto a single correlation curve.

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